



## AP<sup>®</sup> Statistics 2010 Scoring Guidelines Form B

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Question 1

**Intent of Question**

The primary goals of this question were to assess students' ability to (1) compare three distributions of a quantitative variable; (2) construct a stemplot; (3) recognize that different graphical displays of the same data can reveal different characteristics of a distribution.

**Solution**

**Part (a):**

Comparing the medians reveals that the concentration of aldrin tends to be highest for River X and lowest for River Z. About 50 percent of the concentrations of aldrin for Rivers X and Y are higher than all of the concentrations for River Z. River X also displays the most variability in aldrin concentrations, as seen by the largest range and largest IQR, and River Z has the least variability, as judged by both IQR and range. The shapes of the three distributions differ, in that the distribution appears to be skewed to the right for River X, roughly symmetric for River Y and slightly skewed to the left for River Z.

**Part (b):**

Aldrin concentrations (in ppm) for River X  
Leaf unit = 0.1 (for example, 3 | 4 represents 3.4 ppm)

3		47
4		0236678
5		13356
6		
7		35
8		0267

**Part (c):**

The stemplot shows a gap in the distribution of aldrin concentrations for River X between the values of 5.6 and 7.3 ppm of aldrin. This gap is not apparent in the boxplot.

**Scoring**

Parts (a), (b) and (c) are each scored as essentially correct (E), partially correct (P) or incorrect (I).

**Part (a)** is scored as follows:

Essentially correct (E) if the response correctly describes, in context, the center, spread and shape (all three characteristics) of the three distributions *AND* makes a *comparative* statement involving all three distributions for at least one characteristic. Specific numerical values are not required.

  
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**Question 1 (continued)**

Partially correct (P) if the response does not warrant an E, but it includes all three of the following components:

1. Mentions all three distributions
2. Correctly describes at least two of the characteristics (center, spread, shape) of at least two distributions
3. Includes a correct comparison of at least two distributions for at least one characteristic

*OR*

if the response describes all three characteristics of the three distributions but does not make a *comparison* across distributions.

Incorrect (I) otherwise.

*Note:* Context is required to earn an E but not to earn a P.

**Part (b)** is scored as follows:

Essentially correct (E) if a reasonable stemplot that includes a leaf unit key is provided. It is *not* necessary for the key to include measurement units (ppm).

Partially correct (P) if a reasonable stemplot without a leaf unit key is provided.

Incorrect (I) if an unreasonable stemplot or a graph other than a stemplot is provided.

**Part (c)** is scored as follows:

Essentially correct (E) if the response includes a recognition of the gap in the stemplot *AND* gives an indication of where the gap occurs, *OR* if the response comments on bimodality *AND* specifies where the modes/clusters occur.

Partially correct (P) if the response indicates there is a gap or bimodality in the stemplot but does not give an indication of where the gap occurs.

Incorrect (I) otherwise. For example, the response might indicate that the numerical values can be seen in the stemplot but not the boxplot, or that the mean and standard deviation can be computed with the stemplot but not the boxplot, or only that the distribution is skewed to the right.

*Note:* The scoring system counts part (a) at *double weight*. In other words, an E counts as 2 points in part (a) and as 1 point in each of parts (b) and (c). Similarly, a P counts as 1 point in part (a) and as  $\frac{1}{2}$  point in parts (b) and (c).

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**Question 1 (continued)**

- 4 Complete Response**
- 3 Substantial Response**
- 2 Developing Response**
- 1 Minimal Response**

If a response is between two scores (for example,  $2\frac{1}{2}$  points), use a holistic approach to determine whether to score up or down, depending on the overall strength of the response and communication.



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Question 2

**Intent of Question**

The primary goals of this question were to assess students' ability to (1) describe a simple random sampling procedure; (2) identify an effective stratification variable; (3) provide a statistical advantage of a stratified random sample over a simple random sample in context.

**Solution**

**Part (a):**

The administrators could number an alphabetical list of students from 1 to 2,500. They could then use a random number generator from a calculator or computer to generate 200 unique random integers from 1 to 2,500. The students corresponding to those 200 numbers would be asked to participate in the survey.

**Part (b):**

One possible stratification variable might be the school level of the student (elementary, middle, high school). The students' perceptions of the importance of good nutrition in food served may differ depending on the students' ages and therefore on school levels. For example, there may be a difference between what elementary students value in food served as opposed to middle school and high school students.

**Part (c):**

One statistical advantage of using stratified random sampling as opposed to simple random sampling is, for example, if the elementary, middle and high school strata create groups that differ with respect to what they value — and are therefore more homogeneous with respect to opinion on this issue — then for the same overall sample size a more accurate estimate of the overall proportion of students who are satisfied with the food under this contract may result. Another advantage is that stratified random sampling guarantees that each of the school-level strata will have some representation, because it is possible that a simple random sample would miss one or more of the strata completely.

**Scoring**

Parts (a), (b) and (c) are each scored as essentially correct (E), partially correct (P) or incorrect (I).

**Part (a)** is scored as follows:

Essentially correct (E) if the response describes a sampling procedure for generating a simple random sample and the description of the sampling procedure includes sufficient detail for implementation.

Partially correct (P) if random selection is used correctly for generating a simple random sample, but the description of the sampling procedure does not provide sufficient detail for implementation.

Incorrect (I) if random selection is not used in a correct way for a simple random sample.

*Note:* A response in which objects are placed into a hat or a box and then drawn out can only earn an "E" if the response explicitly states that the objects are mixed or that they are drawn out at random.

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Question 2 (continued)

Part (b) is scored as follows:

Essentially correct (E) if the response identifies a reasonable stratification variable and provides a reasonable justification in context (such as stating, “the groups (strata) might differ *with respect to food preferences or nutritional awareness*”).

Partially correct (P) if the response identifies a reasonable stratification variable but provides a weak justification (such as stating only, “the groups (strata) differ”).

Incorrect (I) if the response identifies an unreasonable stratification variable, or provides an unreasonable justification or no justification.

Part (c) is scored as follows:

Essentially correct (E) if the response provides a reasonable statistical advantage of stratified random sampling that is not also true of random sampling, and that is clearly communicated and in context.

Partially correct (P) if the response provides a reasonable statistical advantage that is either not well communicated or that is not in context.

Incorrect (I) if the response includes only a vague potential statistical advantage, such as “data more accurate” or “stratified random sampling is better.”

*Note:* Responses to part (c) such as “stratified random sampling allows for inferences to be drawn for the three grade levels separately about the feelings of students in those grade levels” should be considered incorrect unless also accompanied by a statistical advantage specific to stratified random sampling.

**4 Complete Response**

All three parts essentially correct

**3 Substantial Response**

Two parts essentially correct and one part partially correct

**2 Developing Response**

Two parts essentially correct and one part incorrect

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

**1 Minimal Response**

One part essentially correct and two parts incorrect

OR

Two parts partially correct and one part incorrect

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Question 3

**Intent of Question**

The primary goals of this question were to assess students' ability to (1) recognize binomial distribution scenarios and calculate relevant binomial probabilities; (2) calculate expected values based on the binomial distribution and properties of expectation.

**Solution**

**Part (a):**

Let  $X$  denote the number of correct guesses, assuming that a student guesses randomly among the five options on all 25 questions. Then  $X$  has a binomial probability distribution with  $n = 25$  and

$$p = \frac{1}{5} = 0.20.$$

**Part (b):**

Let  $Y$  denote the number of correct responses on the seven questions for which the student guesses randomly from among the five options. Then  $Y$  has a binomial probability distribution with  $n = 7$  and  $p = 0.20$ . Then the expected value of  $Y$ ,  $E(Y) = np = 7(0.20) = 1.4$  correct responses.

Next, using the scoring formula provided,  
Score =  $(18 + Y) - 0.25(7 - Y) + 0(0) = 16.25 + 1.25Y$ .

The expected exam score is therefore:

$E(\text{Score}) = E(16.25 + 1.25Y) = 16.25 + 1.25E(Y) = 16.25 + 1.25(1.4) = 16.25 + 1.75 = 18$  correct responses.

**Part (c):**

Let  $Y$  be defined as in part (b). The student passes when Score  $\geq 20$ , which means that  $16.25 + 1.25Y \geq 20$ , which means that  $Y \geq \frac{20 - 16.25}{1.25} = 3$ . In other words, in order to pass, the student must get three or more correct from the seven questions on which the student guesses.

$Y$  has a binomial probability distribution with  $n = 7$  and  $p = 0.20$ , so

$$P(Y \geq 3) = 1 - P(Y \leq 2) = 1 - \left[ \binom{7}{0} (.2)^0 (.8)^7 + \binom{7}{1} (.2)^1 (.8)^6 + \binom{7}{2} (.2)^2 (.8)^5 \right] = 1 - 0.852 = 0.148.$$

**Scoring**

Parts (a), (b) and (c) are each scored as essentially correct (E), partially correct (P) or incorrect (I).

  
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**Question 3 (continued)**

**Part (a)** is scored as follows:

Essentially correct (E) if the response identifies the correct type of probability distribution (binomial) AND identifies the two parameter values,  $n = 25$  and  $p = \frac{1}{5} = 0.20$ , correctly. There are two components: naming the probability distribution and identifying the two parameter values.

Partially correct (P) if the response correctly identifies only one of the two components (either the name of the probability distribution or the parameter values).

Incorrect (I) if the response neither correctly names the distribution nor identifies both parameter values correctly.

*Note:* Notation such as  $B(25,0.2)$  will be scored as essentially correct (E) for this part.

**Part (b)** is scored as follows:

Essentially correct (E) if the number of trials,  $n$ , and the binomial probability of a success  $p$  for  $Y$  (the number of correct guesses) are used to find the expected value of  $Y$ ,  $E(Y)$ , and if the correct expected exam score is calculated using  $E(Y)$ . There are two components: calculating  $E(Y)$  and calculating the expected exam score.

Partially correct (P) if only one of the two components is correct.

Incorrect (I) if neither component is correct.

**Part (c)** is scored as follows:

Essentially correct (E) if the response specifies that three or more correct guesses are needed and the binomial probability is calculated correctly. There are two components: correctly identifying the required probability and correct calculation of the probability.

Partially correct (P) if only one of the two components is correct.

Incorrect (I) if neither component is correct.

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**Question 3 (continued)**

**4 Complete Response**

All three parts essentially correct

**3 Substantial Response**

Two parts essentially correct and one part partially correct

**2 Developing Response**

Two parts essentially correct and one part incorrect

*OR*

One part essentially correct and one or two parts partially correct

*OR*

Three parts partially correct

**1 Minimal Response**

One part essentially correct and two parts incorrect

*OR*

Two parts partially correct and one part incorrect

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Question 4

**Intent of Question**

The primary goals of this question were to assess students' ability to (1) calculate and interpret a confidence interval for a population proportion; (2) recognize that it is still reasonable to use the confidence interval procedure even though sampling is without replacement as long as the sample size is small relative to the population size.

**Solution**

**Part (a):**

The sample proportion of songs that were loaded by Lori is  $\hat{p} = \frac{13}{50} = 0.26$ . The conditions for constructing a confidence interval are satisfied because: (1) the problem states that the 50 songs in the sample were randomly selected, and (2)  $n \times \hat{p} = 13$  and  $n \times (1 - \hat{p}) = 37$  are both at least 10. A 90 percent confidence interval for the population proportion  $p$ , the actual proportion of all songs on the player that were loaded by Lori, is:

$$0.26 \pm 1.645 \sqrt{\frac{.26 \times (1 - .26)}{50}} = 0.26 \pm 0.102 = (0.158, 0.362).$$

We can be 90 percent confident that for the population of all songs on the digital music player, the proportion of songs that were loaded by Lori is between 0.158 and 0.362.

**Part (b):**

The sample size of 50 is quite small compared with the population size of 2,384. The usual criterion for checking whether one can disregard the distinction between sampling with or without replacement is to check whether the ratio of the population size to the sample size is large, such as at least 10 or at least 20. In this case the ratio is  $\frac{2,384}{50} = 47.7$ , so the criterion is clearly met, and the confidence interval procedure in part (a) is valid.

**Scoring**

This question is scored in four sections. Part (a) has three components: (1) stating the appropriate confidence interval procedure and checking its conditions; (2) construction of the confidence interval; (3) interpretation of the confidence interval. Section 1 consists of part (a), component 1; section 2 consists of part (a), component 2; section 3 consists of part (a), component 3. Section 4 consists of part (b). Each of the four sections is scored as essentially correct (E), partially correct (P) or incorrect (I).

**Section 1** is scored as follows:

Essentially correct (E) if the response identifies a one-sample z-interval for a proportion (either by name or by formula) and also includes a statement of the random sampling condition and a statement of, and check of, the sample size condition.

Partially correct (P) if the response identifies the correct procedure but adequately addresses only one of the two conditions (random sampling, sample size) OR does not identify the correct procedure but adequately addresses both conditions.

  
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**Question 4 (continued)**

Incorrect (I) if the response identifies the correct procedure but does not adequately address either condition *OR* does not identify the correct procedure and adequately addresses, at most, one condition.

*Notes*

- Stating only that “ $n \times \hat{p}$  and  $n \times (1 - \hat{p})$  are both greater than 10” is only a statement of the sample size condition and is not sufficient for checking it. The response must use specific values from the question in the check of the condition.
- If a response includes an inappropriate condition, such as requiring that  $n \geq 30$  or requiring a normal population, then the response can earn no more than a P for part (a). However, stating and checking a condition about the size of the sample relative to the size of the population is not required but is also not inappropriate.

**Section 2** is scored as follows:

Essentially correct (E) if the response makes use of the appropriate confidence interval procedure and calculates the 90 percent confidence interval correctly.

Partially correct (P) if the response makes use of the appropriate confidence interval procedure but does not include a correct calculation of the 90 percent confidence interval.

Incorrect (I) if the response makes use of an incorrect procedure, such as a  $t$ -interval for a population mean.

**Section 3** is scored as follows:

Essentially correct (E) if the response provides a reasonable interpretation, in context, making clear that the estimate is for the population proportion of songs that were loaded by Lori and that we have 90 percent confidence in the interval.

Partially correct (P) if the response provides a reasonable interpretation, but does not make clear that the estimate is for the population proportion of songs that were loaded by Lori or does not mention 90 percent confidence.

Incorrect (I) if the response provides an incorrect interpretation.

**Section 4** is scored as follows:

Essentially correct (E) if the response states that the difference between sampling with or without replacement is negligible here because the population size is large relative to the sample size *AND* provides a reasonable numerical justification for this assertion.

Partially correct (P) if the response states that the difference between sampling with or without replacement is negligible here because the population size is large relative to the sample size, but provides no numerical justification for this assertion.

Incorrect (I) if the response does not state that the difference between sampling with or without replacement is negligible here because the population size is large relative to the sample size.

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**Question 4 (continued)**

*Notes*

- Reasonable numerical justification includes stating that the sample size is less than 5 percent (or 10 percent) of the population size or that the ratio of the population size to the sample size is greater than 20 (or 10).
- A response that compares the probabilities of songs being selected with or without replacement in terms of the sample and populations sizes, and that concludes that the difference in probabilities will be negligible, may be scored as essentially correct (E).

Each essentially correct (E) section counts as 1 point and each partially correct (P) section counts as  $\frac{1}{2}$  point.

- |          |                             |
|----------|-----------------------------|
| <b>4</b> | <b>Complete Response</b>    |
| <b>3</b> | <b>Substantial Response</b> |
| <b>2</b> | <b>Developing Response</b>  |
| <b>1</b> | <b>Minimal Response</b>     |

If a response is between two scores (for example,  $2\frac{1}{2}$  points), use a holistic approach to determine whether to score up or down, depending on the strength of the response and communication.

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Question 5

**Intent of Question**

The primary goals of this question were to assess students' ability to (1) calculate appropriate probabilities, including conditional probabilities, from a two-way table; (2) determine from a two-way table whether two events are independent; (3) identify an appropriate test procedure for assessing independence between two categorical variables.

**Solution**

**Part (a):**

Using the addition rule, the probability that the randomly selected adult is a college graduate or obtains news primarily from the internet is:

$$\begin{aligned} P(\text{college graduate or internet}) &= P(\text{college graduate}) + P(\text{internet}) - P(\text{college graduate and internet}) \\ &= \frac{693}{2500} + \frac{687}{2500} - \frac{245}{2500} = \frac{1135}{2500} = 0.454. \end{aligned}$$

**Part (b):**

Reading values from the table, the conditional probability that the selected adult receives news primarily from the internet given that he or she is a college graduate is:  $\frac{245}{693} = 0.354$ .

**Part (c):**

These events are not independent. One way to establish this is to note that the unconditional probability equals  $P(\text{obtains news primarily from the internet}) = \frac{687}{2500} = 0.275$ , but the conditional probability equals  $P(\text{obtains news primarily from the internet} / \text{is a college graduate}) = 0.354$ . Because these two probabilities are not equal, the events "is a college graduate" and "obtains news primarily from the internet" are not independent.

**Part (d):**

Chi-square test of association (or independence), with degrees of freedom =  $(\# \text{ of rows} - 1) \times (\# \text{ of columns} - 1) = (5 - 1) \times (3 - 1) = 8$ .

**Scoring**

Parts (a), (b), (c) and (d) are each scored as essentially correct (E), partially correct (P) or incorrect (I).

**Part (a)** is scored as follows:

Essentially correct (E) if the probability is computed correctly and appropriate work is shown *OR* the probability calculation is set up correctly but a minor computational error is made.

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Question 5 (continued)

Partially correct (P) if the probabilities of the two events are added without subtracting the probability of their intersection, resulting in  $\frac{1380}{2500} = 0.552$ .

OR

Independence is assumed in computing the probability of the intersection.

Incorrect (I) if the response does not meet the criteria for an E or P, or includes the correct decimal answer with no accompanying work or justification.

Note: An answer of  $\frac{1135}{2500}$  in fraction form is sufficient to be scored as essentially correct (E).

Part (b) is scored as follows:

Essentially correct (E) if the conditional probability is correctly computed and appropriate work is included OR if the calculation is set up correctly but a minor computational error is made.

Partially correct (P) if the reverse conditional probability (being a college graduate given that he or she primarily obtains news from the internet) is computed, resulting in  $\frac{245}{687} = 0.357$ .

Incorrect (I) if the probability of the *intersection* of the two events is computed, resulting in

$$\frac{245}{2500} = 0.098.$$

OR

The *unconditional* probability of obtaining news primarily from the internet is computed, resulting in

$$\frac{687}{2500} = 0.275.$$

OR

The response otherwise fails to meet the requirements for an E or P.

Notes

- An answer of  $\frac{245}{693}$  in fraction form is sufficient to be scored as essentially correct (E).
- A correct decimal answer with no work or justification is scored as incorrect (I).

Part (c) is scored as follows:

Essentially correct (E) if the response states that the events are not independent and gives a correct numerical justification based on the table.

Partially correct (P) if the response states that the events are not independent and gives a correct statistical justification, but numerical support is not included (for example, says that  $P(C/I) \neq P(C)$  but never reports either probability) OR the response includes correct and relevant calculations related to independence of these events but reaches an incorrect conclusion that the events *are* independent.

Incorrect (I) if the response states that the events are not independent but the given justification is not based on a correct probability argument OR the response does not reveal an understanding of how to assess whether two events are independent by comparing appropriate probabilities.

  
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**Question 5 (continued)**

**Part (d)** is scored as follows:

Essentially correct (E) if the chi-square test of association (or independence) is correctly identified and the correct degrees of freedom are given.

*Note:* It is not necessary to show work in calculating the degrees of freedom.

Partially correct (P) if the response includes the correct name (chi-square test of association or independence) but not the correct degrees of freedom.

Incorrect (I) if the response includes neither identification of the chi-square test of association or independence nor correct degrees of freedom.

*Notes*

- If the response includes only “chi-square test” without specifying “of association (or independence),” this part is scored as essentially correct (E) provided that the degrees of freedom are computed correctly but as incorrect (I) if the degrees of freedom are incorrect.
- If the response identifies the test as “chi-square test of goodness-of-fit” or “chi-square test of homogeneity of proportions,” the response is scored as incorrect (I).
- If the response does not name a correct test and only gives correct degrees of freedom, the response is scored as incorrect (I).

Each essentially correct (E) part counts as 1 point. Each partially correct (P) part counts as  $\frac{1}{2}$  point.

- |          |                             |
|----------|-----------------------------|
| <b>4</b> | <b>Complete Response</b>    |
| <b>3</b> | <b>Substantial Response</b> |
| <b>2</b> | <b>Developing Response</b>  |
| <b>1</b> | <b>Minimal Response</b>     |

If a response is between two scores (for example,  $2\frac{1}{2}$  points), use a holistic approach to determine whether to score up or down, depending on the strength of the response and communication. Also use the following guidelines:

- If part (a) was scored as partially correct (P), always score down.
- A holistic score of 1 may be given to a response with all four parts scored as incorrect (I), if parts (a) and (b) both provide correct decimal answers but received no credit because supporting work was not included.

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Question 6

**Intent of Question**

The primary goals of this investigative task were to assess students' ability to understand, apply and draw conclusions from a regression analysis beyond what they have previously studied. More specific goals were to assess students' ability to (1) interpret a slope coefficient and residual value; (2) interpret a confidence interval; (3) compare two regression models and draw appropriate conclusions.

**Solution**

**Part (a):**

The slope coefficient is 0.165. This means that for each additional square foot of size, the predicted price of the house increases by 0.165 thousand dollars, which is \$165. In other words, this model predicts that the average price of a house increases by \$165 for each additional square foot of a house's size.

**Part (b):**

The residual value of 49 for this house indicates that its actual price is 49 thousand dollars higher than the model would predict for a house of its size.

**Part (c):**

The average residual value for the eight houses with a swimming pool is:

$$\frac{(6 + 49 + (-18) + 42 + 1 + 50 + 9 + (-23) + 42)}{8} = \frac{149}{8} = 18.6 \text{ thousand dollars.}$$

The average residual value for the 17 houses with no swimming pool is:

$$\frac{(13 + 26 + (-45) + \dots + (-58) + (-52) + 33)}{17} = \frac{-150}{17} = -8.8 \text{ thousand dollars.}$$

The residual averages suggest that the regression line tends to underestimate the price of homes with a swimming pool by about 18.6 thousand dollars and to overestimate the price of homes with no pool by about 8.8 thousand dollars. The difference between these two residual averages is  $18.6 - (-8.8) = 27.4$  thousand dollars. This suggests that, for two houses of the same size, the house with a swimming pool would be estimated to cost \$27,400 more than the house with no swimming pool.

**Part (d):**

No, this confidence interval does *not* indicate a significant difference (at the 95 percent confidence level, equivalent to the 5 percent significance level) between the two slope coefficients because the interval includes the value zero.

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Question 6 (continued)

Part (e):

If the two population regression lines do in fact have the same slope, the impact of a swimming pool is the (constant) vertical distance between the two lines. However, because the two fitted lines do not have the same slope, the distance between the two fitted lines depends on the size of the house. Using the available information, there are two acceptable approaches to estimating the impact of having a swimming pool.

Approach 1: Use the two fitted lines to predict the price of a house with and without a pool for a particular house size. For example, using the value of size = 2,250 square feet (which is near the middle of the distribution of house sizes), we find:

Predicted price for a 2,250 square-foot house with a swimming pool =  
 $-11.602 + 0.166 \times 2,250 = 361.898$  thousand dollars.

Predicted price for a 2,250 square-foot house with no swimming pool =  
 $-27.382 + 0.160 \times 2,250 = 332.618$  thousand dollars.

The difference in these predicted prices is  $361.898 - 332.618 = 29.280$  thousand dollars, which is an estimate of the impact of a swimming pool on the predicted price of a 2,250 square-foot house. This is quite similar to the estimate based on residuals in part (c).

Approach 2: Because the slopes of the two sample regression lines were judged not to be significantly different, another acceptable approach would be to use the difference in the intercepts of the two fitted lines as an estimate of the vertical distance between the two population regression lines.

The difference in the intercepts of the two fitted lines is  $-11.602 - (-27.382) = 15.780$  thousand dollars, which is an estimate of the impact of a swimming pool on the predicted price of a house, assuming this difference does not change with the size of the house. This is quite different from the estimate based on residuals in part (c).

**Scoring**

This question is scored in four sections. Section 1 consists of part (a); section 2 consists of part (b); section 3 consists of part (c); section 4 consists of parts (d) and (e). Each of the four sections is scored as essentially correct (E), partially correct (P) or incorrect (I).

**Section 1** is scored as follows:

Essentially correct (E) if the response identifies the correct value for the slope coefficient and provides a correct interpretation in context.

Partially correct (P) if the response identifies the correct value for the slope coefficient and provides a correct interpretation but not in context *OR* the response provides an incorrect value for the slope but provides a correct interpretation of this value in context *OR* the response identifies the correct value for the slope but the interpretation is incomplete because of one or more of the following errors:

- The interpretation does not mention “predicted” or “on average” or any other indication of a probabilistic rather than a deterministic relationship.

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Question 6 (continued)

- The interpretation does not include the notion of each *additional* square foot of size by saying something like “for every square foot.”
- The interpretation does not use units for the price variable, or it uses incorrect units for the price variable (e.g., dollars instead of thousands of dollars).

Incorrect (I) if there is no interpretation or if the interpretation does not warrant a score of P.

*Note:* It is possible to earn an E for section 1 without stating the actual numerical value of the slope, if a correct and well-communicated interpretation of the slope is given in context.

**Section 2** is scored as follows:

Essentially correct (E) if the response provides a correct interpretation of the residual value, in context, including both direction and a comparison with the model’s predicted or average value (e.g., actual price is higher than predicted).

Partially correct (P) if the response provides an interpretation of the residual value that fails to mention direction or that gives the incorrect direction *OR* if the response provides a correct interpretation of the residual value that includes direction, but that is not in context.

Incorrect (I) if there is no interpretation of the residual value *OR* the interpretation does not include direction and is not in context.

**Section 3** is scored as follows:

Essentially correct (E) if the response correctly calculates averages of residual values both for houses with pools and houses without pools *AND* correctly reports the difference between those averages as the estimate of the impact of a swimming pool.

Partially correct (P) if the response either correctly calculates averages of residual values both for houses with pools and houses without pools but does not correctly report the difference between those averages as the estimate of the impact of a swimming pool *OR* incorrectly calculates one or both averages of residual values but does report the difference between those averages as the estimate of the impact of a swimming pool *OR* does not use all of the residual values but does use a reasonable set of residual values (such as houses of similar size) and correctly calculates both averages and correctly reports the difference between those averages as the estimate of the impact of a swimming pool.

Incorrect (I) if the response does not meet the criteria for an E or P.

*Notes:*

- If the student calculates some other measure of center for the two sets of residuals (e.g., medians) and reports the difference as the estimate of the impact of a swimming pool, this part can be scored, at best, partially correct (P).
- If the student estimates the values of the residuals from the residual plot rather than using the residuals provided in the table, the response can be scored as essentially correct (E), provided it is clear that this is what was done.

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Question 6 (continued)

Section 4 is scored as follows:

Essentially correct (E) if the response includes all three of the following components:

1. Correctly notes that the confidence interval in part (d) includes zero and so the difference in the slopes is not statistically significant.
2. Calculates a reasonable estimate in part (e):
  - For approach 1, this includes choosing a house size within the range of the data and correctly computing the difference in predicted prices.
  - For approach 2, this includes appealing to the fact that the slopes were judged as not significantly different and computing the difference in intercepts.
3. Includes a comparison of the estimate in part (e) to the estimate in part (c).

Partially correct (P) if the response includes only one of (1) and (2) above.

Incorrect (I) if the response includes neither (1) nor (2) above.

*Notes*

- If the response uses approach 1, the difference between the two predicted values can range from 25.38 to 33.44, depending on the house size used.
- If the response uses approach 2, the constant vertical distance can be estimated from the graph showing the two regression lines rather than on the difference in intercepts, provided that the response makes it clear that this is what is being done.
- In the comparison with the estimate in part (c), an assessment of the size of the difference in estimates is not required. Statements that merely use phrases like “greater than,” “about the same,” etc. are acceptable for the comparison component of parts (d) and (e).
- If this section receives a score of partially correct only because the student neglects to compare the estimate in part (e) to the estimate in part (c), the response should be scored up if a decision on whether to score up or down is required.
- If the response subtracts the two fitted equations to obtain a general expression for the vertical distance between the two fitted lines as a function of house size, this should be considered an essentially correct approach for component 2 of section 4. The resulting expression is  $15.580 + 0.006 \cdot (\text{size})$ .
- If the student uses a house size outside the range of the data to compute the difference in predicted price, this can only be considered correct if the student appeals to the fact that the slopes of the sample regression lines are not significantly different.

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**Question 6 (continued)**

Each essentially correct (E) section counts as 1 point. Each partially correct (P) section counts as  $\frac{1}{2}$  point.

- 4            Complete Response**
- 3            Substantial Response**
- 2            Developing Response**
- 1            Minimal Response**

If a response is between two scores (for example,  $2\frac{1}{2}$  points), use a holistic approach to determine whether to score up or down, depending on the overall strength of the response and communication. In deciding whether to score up or down, pay particular attention to the response to the investigative part of the question (section 4).

